The Money Value of a Man

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Common View: Most valuable asset that most people hold is their own human capital.

Questions:

What are the properties of the value of an individual's human capital?

What are the properties of the associated returns?

Value and Return Concepts:

$$v_j \equiv E_j[\sum_{k=j+1}^{J} m_{j,k} e_k]$$

$$R^h_{j+1} \equiv \frac{v_{j+1} + e_{j+1}}{v_j}$$

What we do: (1) justify this notion of value and (2) analyze (v_j, R^h_j) using US data .

Motivation:

1. Portfolio advice.

2. International portfolio diversification puzzle

3. Welfare Gains of moving to a smoother consumption plan

4. Estimating preference parameters

Preview of our Findings

1. Value

- Far below the value of discounting future earnings at the risk-free rate

- Stock component is smaller than the bond component

- Large negative orthogonal component early in life

2. Returns

- Mean returns very large early in life and decline with age

- Small positive correlation with stock return

Related Literature:

1. Value of Human Capital

- Farr (1853), Weisbrod (1961), Becker (1975), ... discount earnings at a deterministic rate. Not useful for analyzing returns.

2. Return to Human Capital

- Campbell (1996), Baxter and Jermann (1997), ... use aggregate earnings data

- Huggett and Kaplan (2011) put bounds on values/returns.

- Mincerian Returns literature focuses on a different notion of a return.

3. Portfolio Allocation

- Campbell et al (2001), Benzoni, Collin-Dufresne and Goldstein (2007), Lynch and Tan (2011).

- focus is on understanding what impacts portfolio composition

Justification for the Notion of Value:

Problem P1

$\max U(c,n)$ where $c = (c_1, .., c_J)$ and $n = (n_1, .., n_J)$

1 $c_j + \sum_{i \in \mathcal{I}} a^i_{j+1} = e_j + \sum_{i \in \mathcal{I}} a^i_j R^i_j$

2 $e_j = G_j(y^j, n^j, z^j)$, $0 \le n_j \le 1$ and $a^i_{J+1} \ge 0$

Important: U is concave

Unimportant: Restrictions on G, number of assets, ...

Problem P2

max $U(c, n)$

1 $c_j + \sum_{i \in \mathcal{I}} a^i_{j+1} + s_{j+1} v_j + p_j n_j = s_j(v_j + d_j) + \sum_{i \in \mathcal{I}} a^i_j R^i_j$

2 $0 \leq n_j \leq 1$ and $a^i_{J+1} \geq 0$

Personalized prices: (p_j, v_j)

Definitions:

1. $m_{j,k}(z^k) = \frac{dU(c^*,n^*)/dc_k(z^k)}{dU(c^*,n^*)/dc_j(z^j)}\frac{1}{P(z^k|z^j)}$ - stochastic discount factor

2. $v_j(z^j) = E[\sum_{k=j+1}^{J} m_{j,k}d_k|z^j]$ - value of human capital

3. $d_j(z^j) = e_j^*(z^j) + p_j(z^j)n_j^*(z^j)$ - dividends

4. $p_j(z^j) = \frac{dU(c^*,n^*)/dn_j(z^j)}{dU(c^*,n^*)/dc_j(z^j)}$ - price of leisure

Theorem: *If* $(c^*, n^*, e^*, y^*, a^*)$ *solves P1 and* c^* *is strictly positive, then* (c^*, n^*, a^*, s^*) *solves P2, where* $s_j^* = 1$*, when the agent takes* (v_j, d_j, p_j) *as given.*

Intuition : P2 is a concave program. Necessary conditions to P1 are sufficient for P2.

Extensions: value earnings and leisure components separately or allow financials frictions

Naive Value: $v_j^{naive} = E[\sum_{k=j+1}^{J} \frac{1}{(1+r)^{k-j}} d_k | z^j]$

Our Notion of Value: $v_j = E[\sum_{k=j+1}^{J} m_{j,k} d_k | z^j]$

When will the two notions differ?

$$v_j = \sum_{k=j+1}^{J} E[m_{j,k}|z^j]E[d_k|z^j] + \sum_{k=j+1}^{J} cov(m_{j,k}, d_k|z^j)$$

When: $E[m_{j,k}|z^j] < 1/(1+r)^{k-j}$ agent is on a corner

When: $cov(m_{j,k}, d_k|z^j) < 0$

Simple Example

max $E[\sum_{j=1}^{J} \beta^{j-1} u(c_j) | z^1]$ subject to

$$(1)\ c_j + a_{j+1} \leq a_j(1+r) + e_j, (2)\ c_j \geq 0, a_{J+1} \geq 0$$

$u(c) = \frac{c^{1-\rho}}{(1-\rho)}$ - CRRA

$e_j = \prod_{k=1}^{j} z_k$ and $\ln z_k \sim N(\mu, \sigma^2)$ i.i.d.

$(1+r) > 0$ - one risk-free asset

Theorem: *If* $1+r = \frac{1}{\beta}exp(\rho\mu - \frac{\rho^2\sigma^2}{2})$ *and initial assets are zero, then* $(c_j, a_{j+1}) = (e_j, 0), \forall j$ *solves the decision problem. Furthermore, at this solution:*

(i) the value of human capital is $v_j(z^j) = f_j e_j$,

$f_j = \sum_{k=j+1}^{J} \beta^{k-j} exp((k-j)[(-\rho+1)\mu + (-\rho+1)^2 \frac{\sigma^2}{2}])$.

(ii) the return to human capital satisfies:

(a) $R^h_{j+1} = (\frac{1+f_{j+1}}{f_j}) z_{j+1}$

(b) $E[R^h_{j+1}|z^j] = \frac{1}{\beta}exp(\mu\rho + \frac{\sigma^2}{2}(1-(1-\rho)^2))$

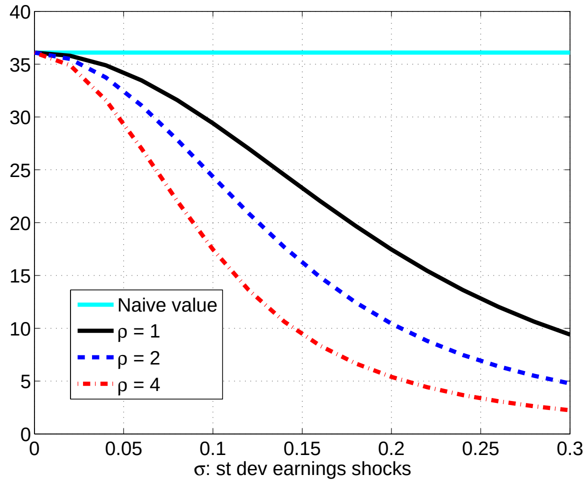
Quantitative Analysis:

Model Periods: age $20-65$

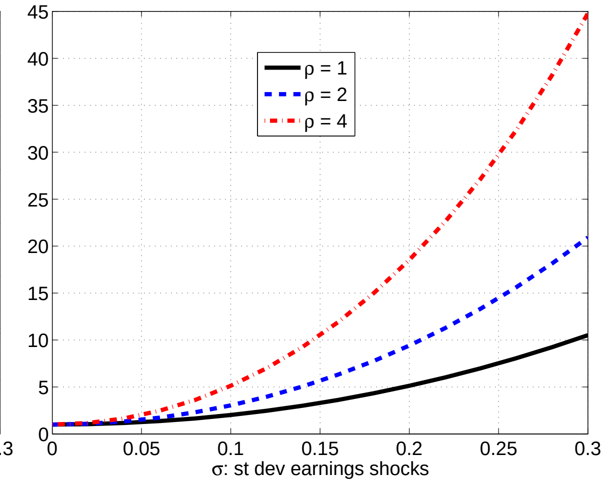
Flat Earnings Profile: $e_1 = 1$ and $\mu = -\sigma^2/2$ and $\sigma \in [0, .3]$

Risk Aversion Parameter: $\rho \in \{1, 2, 4\}$

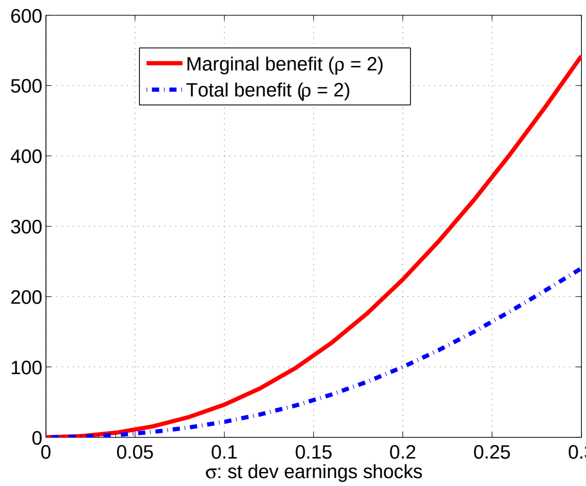
Interest Rate: $1.01 = 1 + r = \frac{1}{\beta} exp(\rho\mu - \frac{\rho^2\sigma^2}{2})$



(a) Value of human capital



(b) Mean return human capital (%)



(c) Benefit of moving to a smooth consumption plan (%)

Figure 1: Human capital values and returns: simple example

Notes:

Marginal Welfare Gains - Alvarez and Jermann (2004)

$$U((1+\Omega(\alpha))c) = U((1-\alpha)c + \alpha c^{smooth})$$

$$\Omega'(0) = \frac{\sum_{j=1}^{J} \sum_{z^j} \frac{dU(c)}{dc_j(z^j)}(c_j^{smooth}(z^j) - c_j(z^j))}{\sum_{j=1}^{J} \sum_{z^j} \frac{dU(c)}{dc_j(z^j)} c_j(z^j)}$$

$$\Omega'(0) = \frac{E[\sum_{j=1}^{J} m_{1,j} c_j^{smooth} | z^1]}{v_1^e(z^1) + e_1(z^1) + \sum_{i \in \mathcal{I}} a_1^i(z^1) R^i(z^1)} - 1$$

Benchmark Model:

$U(c_1, ..., c_J) = W(c_1, F(U(c_2, ..., c_J)); j)$ - Epstein-Zin utility

$W(a, b; j) = [(1-\beta)a^{1-\rho} + \beta\psi_{j+1}b^{1-\rho}]^{1/(1-\rho)}$

$F(x) = (E[x^{1-\alpha}])^{1/(1-\alpha)}$

Two assets: bond and stock

Exogenous earnings: $e_j = G_j(z^j)$

Extra restrictions: $a^s_{j+1} \geq 0 \ and \ a^s_{j+1} \leq p(a^s_{j+1} + a^b_{j+1})$

Empirical Framework I:

$\log e_{i,j,t} = u_t^1 + u_{i,j,t}^2$

$u_{i,j,t}^2 = \alpha^i + \kappa_j + \zeta_{i,j,t} + \nu_{i,j,t}$

$\zeta_{i,j,t+1} = \rho\zeta_{i,j,t} + \eta_{i,j,t+1}$ and $\zeta_{i,0,t} = 0$

$\alpha \sim N(0, \sigma_\alpha^2), \eta \sim N(0, \sigma_\eta^2(\Delta u_t^1)), \nu \sim N(0, \sigma_\nu^2(\Delta u_t^1))$

Empirical Framework II: $y_t = (u_t^1, P_t)'$ where $\log R_t^s = \Delta P_t$

$$y_t = v + \sum_{i=1}^{p} A_i y_{t-i} + \varepsilon_t$$

If $y_t \sim I(1)$ can rewrite as VECM ($w_t \equiv \beta' y_t + \mu$)

$$\begin{pmatrix} \Delta y_t \\ w_t \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta'\gamma \end{pmatrix} + \sum_{i=1}^{p-1} \begin{pmatrix} \Gamma_i \\ \beta'\Gamma_i \end{pmatrix} \Delta y_{t-i} + \begin{pmatrix} \alpha \\ 1 + \beta'\alpha \end{pmatrix} w_{t-1} + \begin{pmatrix} \varepsilon_t \\ \beta'\varepsilon_t \end{pmatrix}$$

$$\begin{pmatrix} \Delta y_t \\ w_t \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta'\gamma \end{pmatrix} + \begin{pmatrix} \Gamma & \alpha \\ \beta'\Gamma & 1 + \beta'\alpha \end{pmatrix} \begin{pmatrix} \Delta y_{t-1} \\ w_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \beta'\varepsilon_t \end{pmatrix}$$

Data Sources:

Idiosyncratic Earnings:

- PSID 1967- 1996 (three samples: FULL, HS and COL)

- Male heads age 22-60 w/ annual earnings $> 1,000$

Aggregate Component:

- PSID 1967-96 and CPS 1967-2009 (three samples)

Asset Returns:

Ken French's Data Archive: (1) stock return is NYSE/AMEX/NASDAQ return, (2) bond return is based on 6-month TBill

Table 1: Idiosyncratic Earnings Process

Parameters	Full Sample	College Sub-sample	High School Sub-sample
ρ	0.957	0.959	0.902
σ_α^2	0.110	0.092	0.121
av. σ_η^2	0.033	0.033	0.039
$\sigma_\eta^2(L) - \sigma_\eta^2(H)$	0.020	0.012	0.025
av. σ_ε^2	0.150	0.151	0.147
Linear time trend in σ_η^2	0.0011	0.0014	0.0013

Table 3: Steady-State Statistics for the Aggregate Process

	Full Sample		
	Data	No Cointegration	With Cointegration
$E\left(logR_t^b\right)$	0.012	0.012	0.012
$E\left(\log R_t^s\right)$	0.041	0.045	0.047
$E\left(\Delta u_t^1\right)$	-0.002	-0.004	-0.003
$sd\left(\Delta u_t^1\right)$	0.025	0.025	0.025
$sd\left(\log R_t^s\right)$:	0.187	0.187	0.188
$corr\left(\Delta u_t^1, \log R_t^s\right)$	0.184	0.177	0.172
$corr\left(\Delta u_t^1, \Delta u_{t-1}^1\right)$	0.425	0.441	0.420
$corr\left(\log R_t^s, \log R_{t-1}^s\right)$	0.057	0.055	0.039
$corr\left(\Delta u_t^1 \log R_{t-1}^s\right)$	0.372	0.398	0.390
$corr\left(\log R_t^s, \Delta u_{t-1}^1\right)$	-0.292	-0.270	-0.292

Table 3: Steady-State Statistics for the Aggregate Process

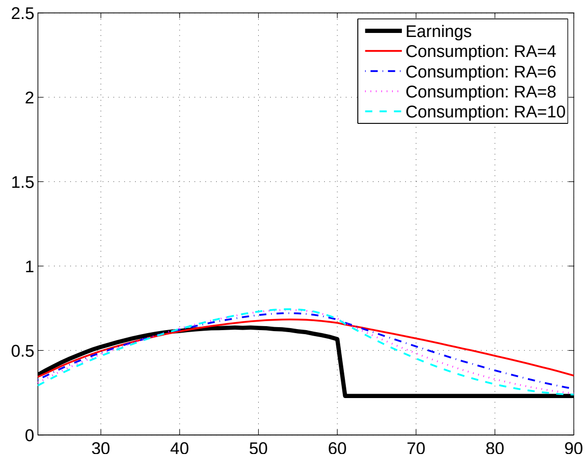
	College Sub-sample		
	Data	No Cointegration	With Cointegration
$E\left(logR_t^b\right)$	0.012	0.012	0.012
$E\left(\log R_t^s\right)$	0.041	0.040	0.045
$E\left(\Delta u_t^1\right)$	0.000	-0.001	-0.001
$sd\left(\Delta u_t^1\right)$	0.023	0.023	0.023
$sd\left(\log R_t^s\right)$:	0.187	0.187	0.186
$corr\left(\Delta u_t^1, \log R_t^s\right)$	0.248	0.251	0.243
$corr\left(\Delta u_t^1, \Delta u_{t-1}^1\right)$	0.346	0.341	0.342
$corr\left(\log R_t^s, \log R_{t-1}^s\right)$	0.057	0.084	0.050
$corr\left(\Delta u_t^1 \log R_{t-1}^s\right)$	0.377	0.387	0.367
$corr\left(\log R_t^s, \Delta u_{t-1}^1\right)$	-0.225	-0.235	-0.229

Table 3: Steady-State Statistics for the Aggregate Process

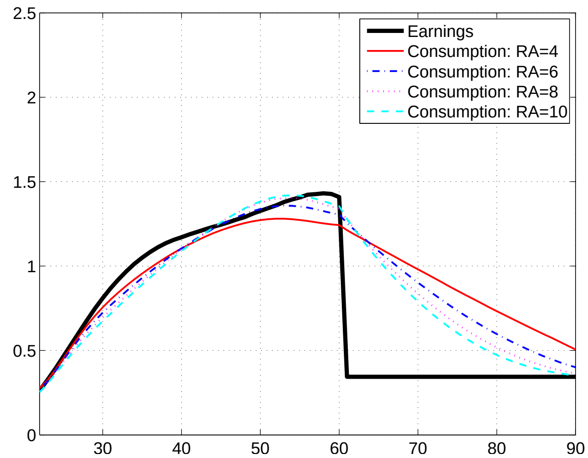
	High School Sub-sample		
	Data	No Cointegration	With Cointegration
$E\left(logR_t^b\right)$	0.012	0.012	0.012
$E\left(\log R_t^s\right)$	0.041	0.045	0.055
$E\left(\Delta u_t^1\right)$	-0.007	-0.010	-0.008
$sd\left(\Delta u_t^1\right)$	0.030	0.030	0.030
$sd\left(\log R_t^s\right)$:	0.187	0.187	0.190
$corr\left(\Delta u_t^1, \log R_t^s\right)$	0.207	0.194	0.192
$corr\left(\Delta u_t^1, \Delta u_{t-1}^1\right)$	0.386	0.416	0.408
$corr\left(\log R_t^s, \log R_{t-1}^s\right)$	0.057	0.047	0.037
$corr\left(\Delta u_t^1 \log R_{t-1}^s\right)$	0.387	0.420	0.420
$corr\left(\log R_t^s, \Delta u_{t-1}^1\right)$	-0.289	-0.261	-0.277

Table 4: Parameter Values for the Benchmark Model

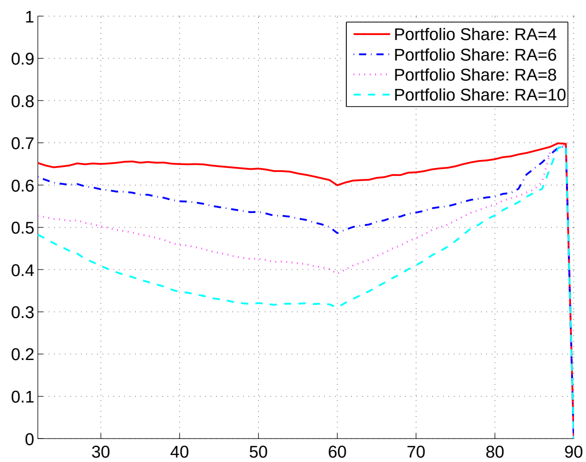
Category	Symbol	Parameter Value
Lifetime	J, Ret	$(J, Ret) = (69, 40)$
Preferences	α Risk Aversion	$\alpha \in \{4, 6, 8, 10\}$
	$1/\rho$ Intertemporal Substitution	$1/\rho = 1.17$
	ψ_{j+1} Survival Probability	U.S. Life Table
	β Discount Factor	see Notes
Earnings	$e_j(z) = z_1 g_j(z_2)(1-\tau)$ $j < Ret$	Table1-2
	$e_j(z) = z_1 b(\alpha)$ $j \geq Ret$	
		$\tau = .27$
		$b(\cdot)$ see text
Returns	R^s, R^b	Table 2 - 3
Leverage	p	$p = 1$



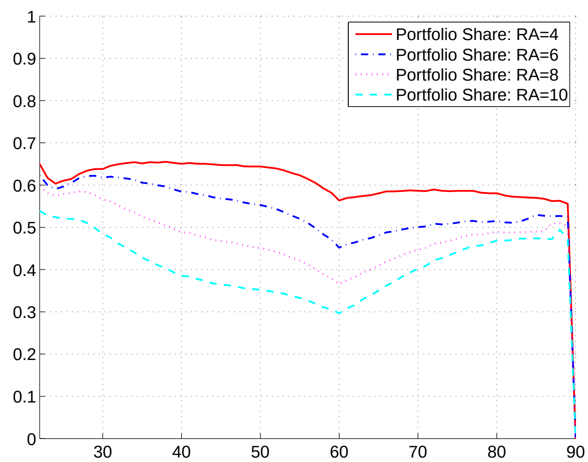
(a) Mean earnings and consumption (high school)



(b) Mean earnings and consumption (college)



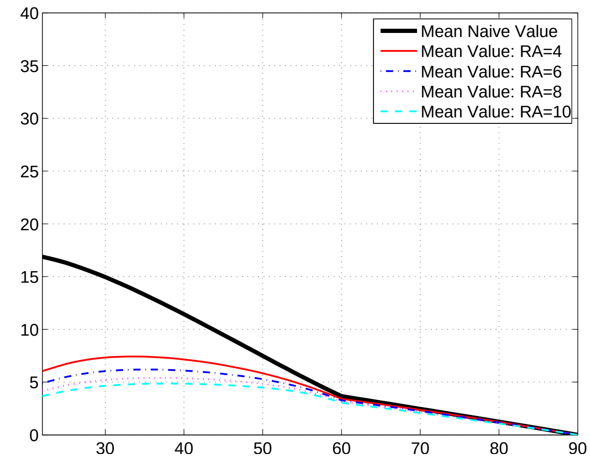
(c) Mean share in equities (high school)



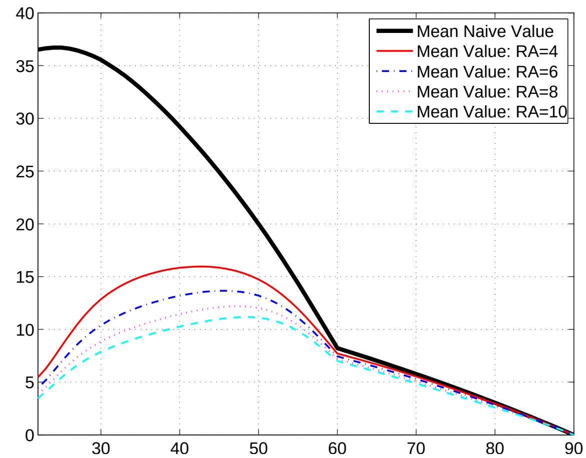
(d) Mean share in equities (college)

Figure 2: Life-cycle profiles in the benchmark model

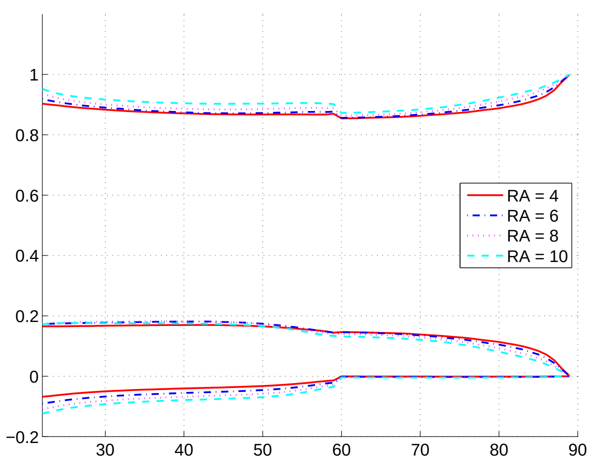
Notes:



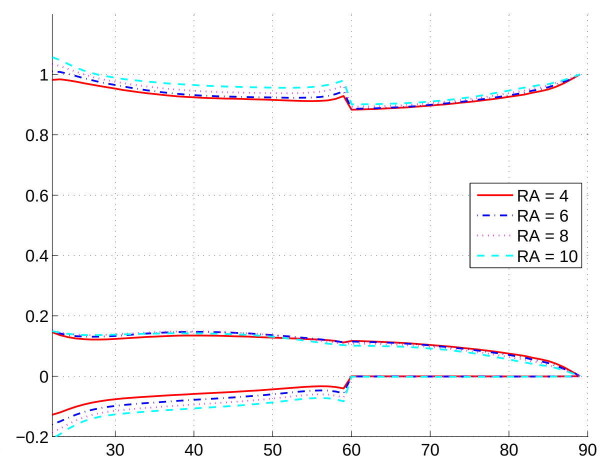
(a) Human capital values (high school)



(b) Human capital values (college)



(c) Decomposition (high school)



(d) Decomposition (college)

Figure 3: Human capital values and decomposition

Notes:

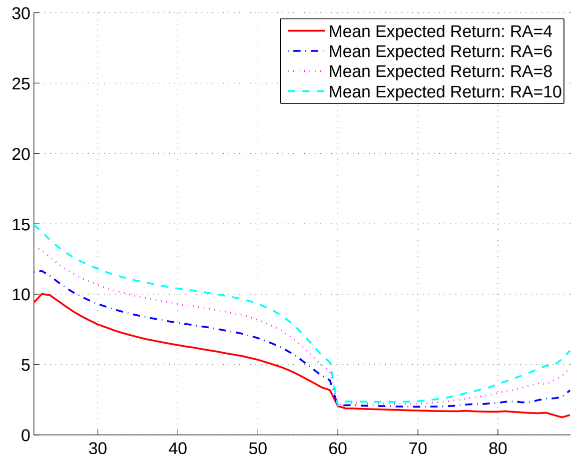
Decomposing Values:

$$v_j = E_j[m_{j,j+1}y_{j+1}] = E_j[m_{j,j+1}(\alpha^b R^b_{j+1} + \alpha^s R^s_{j+1} + \epsilon)]$$

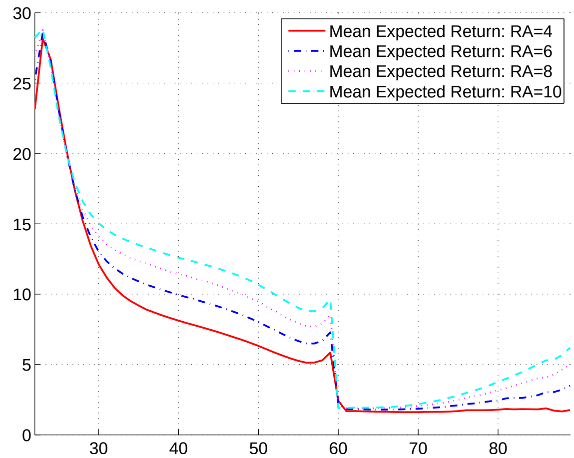
$$v_j = \alpha^b E_j[m_{j,j+1}R^b_{j+1}] + \alpha^s E_j[m_{j,j+1}R^s_{j+1}] + E_j[m_{j,j+1}\epsilon]$$

$$1 = \frac{\alpha^b}{v_j}E_j[m_{j,j+1}R^b_{j+1}] + \frac{\alpha^s}{v_j}E_j[m_{j,j+1}R^s_{j+1}] + \frac{E_j[m_{j,j+1}\epsilon]}{v_j}$$

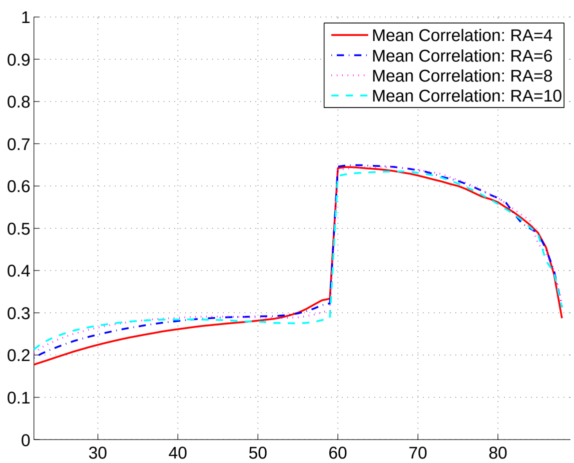
Upshot: decompose v_j into three components. bond and stock shares sum to more than 1 when $E_j[m_{j,j+1}\epsilon] < 0$



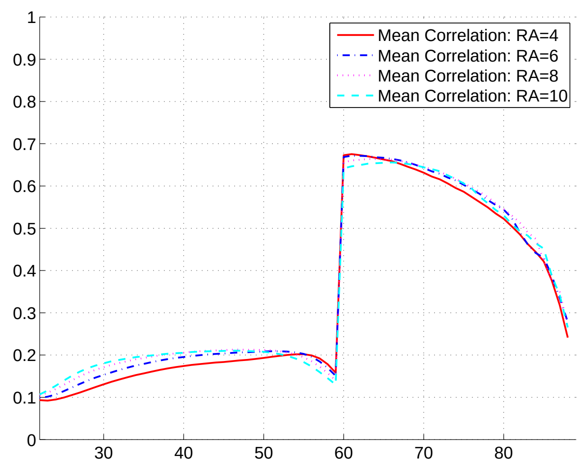
(a) Human capital returns (%) (high school)



(b) Human capital returns (%) (college)



(c) Correlation: HC, stock returns (high school)



(d) Correlation: HC, stock returns (college)

Figure 4: Properties of human capital returns

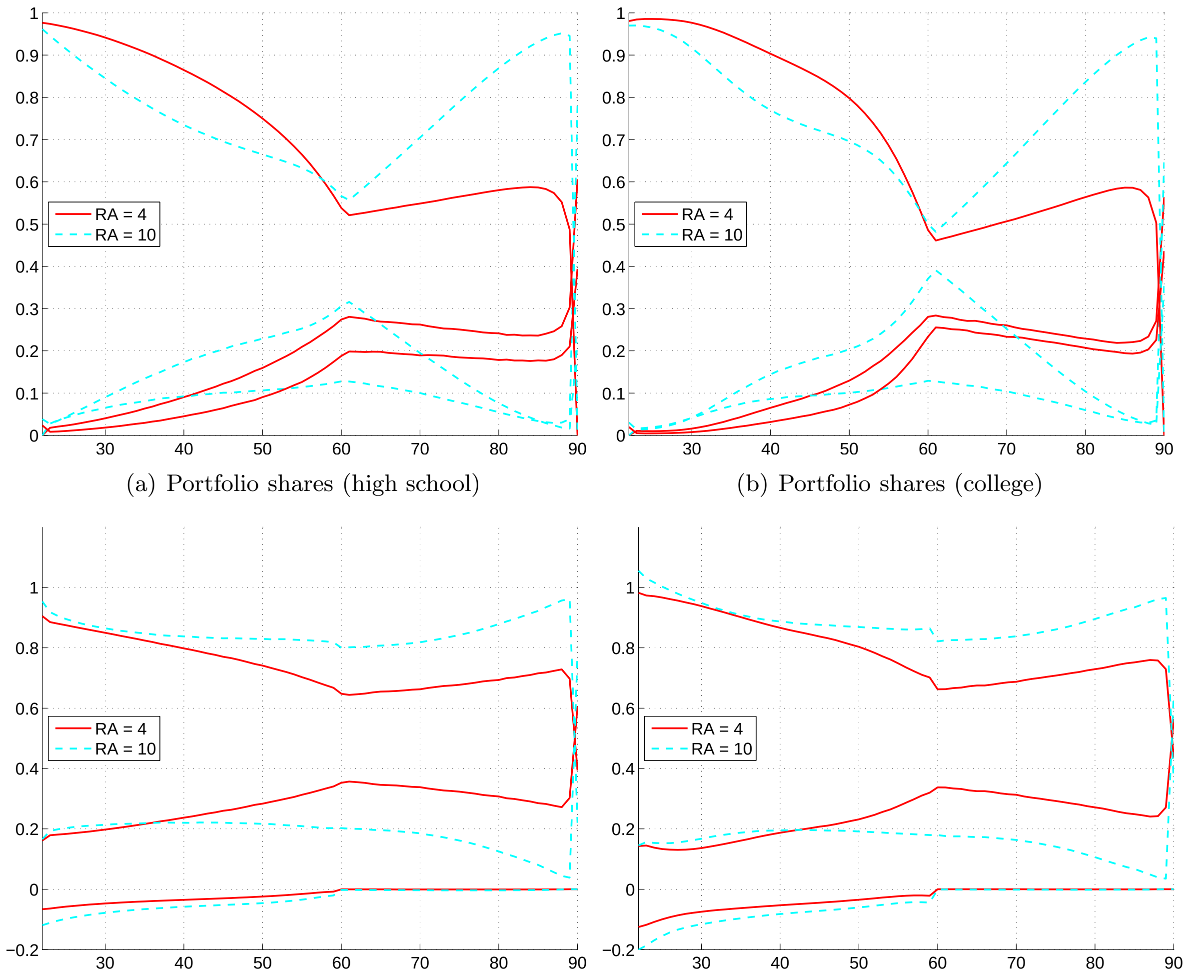
Notes:

Decomposing Returns:

$$R^h_{j+1} \equiv \frac{v_{j+1} + e_{j+1}}{v_j} = \frac{\alpha^b R^b_{j+1} + \alpha^s R^s_{j+1} + \epsilon}{v_j}$$

$$E_j[R^h_{j+1}] = \frac{\alpha^b}{v_j} E_j[R^b_{j+1}] + \frac{\alpha^s}{v_j} E_j[R^s_{j+1}]$$

Upshot: weights sum to more than 1 when $E_j[m_{j,j+1}\epsilon] < 0$



(a) Portfolio shares (high school)

(b) Portfolio shares (college)

(c) Overall portfolio shares (high school)

(d) Overall portfolio shares (college)

Figure 5: Portfolio shares in the benchmark model

Notes:

"Link your analysis to financing human capital investments"

1. Our work does not offer a direct link. Earnings are exogensous. However, some of our findings (e.g. $v_j << v_j^{naive}$ and $E[R_j^h] > E[R_j^s] > E[R_j^b]$) should also hold when earnings are endogenously determined.

2. Such a model (e.g. Huggett, Ventura and Yaron (2011)) would allow marginal returns and total human capital returns to be analyzed jointly.

3. Huggett, Ventura and Yaron (2011) would also suggest that analyzing human capital accumulation and its financing from age 20 or so onwards is way too late. They find that (averaging across individuals) there is little net skill accumulation after this age.